INDUCED PSEUDOSCALAR COUPLING IN MUON CAPTURE AND SECOND-ORDER CORRECTIONS

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Abstract

The hyperfine effect in muon capture rate is analyzed with taking into account of the second-order terms in $1/M$ in the non-relativistic Hamiltonian. It is shown that in the situation of the dominance of the squared first-order terms the interference of zero-order and second-order terms has no significant influence. General expression for neutrino (recoil nucleus) angular distribution in muon capture by nucleus with non-zero spin is proposed. High sensitivity to g_P of the term related with alignment of the initial mesic atom is discussed in the approximation of dominating Gamow–Teller matrix element.

1 Introduction

PCAC hypothesis gives a relation between the induced pseudoscalar form factor and the axial vector one. In muon capture one expects $g_P \simeq 7g_A \simeq -8.4$. This prediction is in agreement with measurements of g_P on proton and some light nuclei as ^{12}C and ^{23}Na (see, e.g., [1]). However, the experiments on muon capture by ^{6}Li [2] and ^{28}Si [3] yielded the values for g_P close to zero. To clarify a situation with this suppression that partly may be due to medium effects further investigations are of interest.

One of the possible ways to measure g_P is the hyperfine effect (see, e.g., [1, 4]). A ratio of muon capture rates Λ_+/Λ_- from different hyperfine sublevels is rather sensitive to the induced pseudoscalar coupling. In particular, if Gamow–Teller matrix element dominates in the transition $|J\pi\rangle \to |J-1\,\pi\rangle$, then the ratio

$$\frac{\Lambda_+}{\Lambda_-} = \frac{G_P^2}{\dfrac{3(2J+1)}{J}\left(G_A - \dfrac{1}{3}G_P\right)^2 + \dfrac{J-1}{3J}G_P^2}, \tag{1}$$

exhibits the high sensitivity to $G_P = (E_\nu/2M)(g_P - g_A - g_V - g_M)$ or to the form factor g_P. Here E_ν is the neutrino energy, and M is the nucleon mass. However, it should be noted that the ratio Λ_+/Λ_- is proportional to $(1/M)^2$, whereas the usually used description of the hyperfine effect [5]-[8] is based on the non-relativistic Hamiltonian which is of first-order in $1/M$. Thus the analysis based on the second-order Hamiltonian seems necessary for consistent description of the hyperfine effect. Such analysis was the first aim of this work.

The second aim was to obtain the general expression for the angular distribution of neutrinos (recoil nuclei) in muon capture by target with non-zero spin J. Due to the hyperfine splitting in the initial mesic atom interference of states with different angular momenta $F_\pm = J \pm 1/2$ gives no contribution to the differential probability of muon

capture. Therefore, the angular distribution of neutrinos is determined by sum

$$\frac{dw(\mathbf{n}_\nu)}{d\Omega} = \sum_F P(F) \frac{dw^F(\mathbf{n}_\nu)}{d\Omega}, \tag{2}$$

where $P(F)$ is the population of the state $|F\rangle$. Thus, irrespective of whether the process of capture from states with different F are detected experimentally or not, the angular distribution of neutrinos must be calculated separately for each state of the hyperfine structure.

Explicit expressions for the asymmetry of neutrino emission along and opposite the direction $\mathbf{n}_\mu$ of mesic atom polarization were obtained in Refs.[9],[5],[7]. However, the ensemble of the initial mesic atoms with spin $F > 1/2$ may not only be polarized but aligned as well. This leads to the anisotropy of neutrino emission in the directions parallel and orthogonal to the vector $\mathbf{n}_\mu$. The alignment effect was analyzed for transitions $|1/2\,\pi\rangle \to |1/2\,\pi\rangle$ and $|1\,\pi\rangle \to |0\,\pi\rangle$ in a model-independent (elementary-particle) approach in Refs.[10, 11] and for transitions $|J\pi\rangle \to |J \pm 1\,\pi\rangle$ in the approximation of dominating Gamow–Teller matrix element in Ref.[12]. It was pointed out that the alignment effect is linear in $1/M$ and is very sensitive to g_P. Nevertheless, the contributions of non-leading matrix elements, in particular, of velocity terms, are evidently of great interest.

2 Second-order Hamiltonian

In the initial 1s-state of mesic atom a captured muon is described by the 4-component wave function. It can be written in the non-relativistic approximation as ($\hbar = c = 1$)

$$\psi_\mu(\sigma_\mu, \mathbf{r}_\mu, t) = \psi_{1s}(r_\mu) \begin{pmatrix} \varphi_\mu(\sigma_\mu) \\ 0 \end{pmatrix} e^{-iE_\mu t}, \tag{3}$$

where E_μ is the total muon energy, including its binding energy in the atom, $\varphi_\mu(\sigma_\mu)$ is the two-component spinor, and σ_μ is the projection of the spin $s_\mu = 1/2$ onto the z axis. A final neutrino with momentum $\mathbf{k}_\nu$ and projection σ_ν of the spin $s_\nu = 1/2$ onto the z axis is described by the wave function

$$\psi_\nu(\sigma_\nu, \mathbf{r}_\nu, t) = u_\nu(\mathbf{k}_\nu, \sigma_\nu) e^{i(\mathbf{k}_\nu \mathbf{r}_\nu - E_\nu t)}, \tag{4}$$

where $u_\nu(\mathbf{k}_\nu, \sigma_\nu)$ is a 4-spinor, and $E_\nu = k_\nu$ is the neutrino energy. Assuming that the weak nucleon-lepton interaction is pointlike, we introduce the lepton current acting on the nucleon

$$j_\lambda(\sigma_\mu, \sigma_\nu) e^{-i(\mathbf{k}_\nu \mathbf{r} + \omega t)} = i\psi_\nu^+(\sigma_\nu, \mathbf{r}, t)\gamma_4(1 + \gamma_5)\psi_\mu(\sigma_\mu, \mathbf{r}, t), \tag{5}$$

where $\omega = E_\mu - E_\nu$. Thus the relativistic Hamiltonian for the nucleon into a nucleus involving in capture of muon being in the spin state $|\sigma_\mu\rangle$ with emission of neutrino in the state $|\mathbf{k}_\nu, \sigma_\nu\rangle$ is

$$\hat{H} = M\beta + \boldsymbol{\alpha}\hat{\mathbf{p}} + U(r) + \hat{H}_W, \tag{6}$$

$$\hat{H}_W = \frac{G\cos\theta_C}{\sqrt{2}} j_\lambda(\sigma_\mu, \sigma_\nu) e^{-i(\mathbf{k}_\nu \mathbf{r}_\nu + \omega t)} (i\Gamma_\lambda)\hat{\tau}_-, \tag{7}$$

where G is the weak-interaction coupling constant, θ_C is the Cabibbo angle. The lowering operator $\hat{\tau}_-$ acts in isospin space and transforms a proton into a neutron. For simplicity

we take the nucleon-nucleus potential $U(r)$ in a central form. The operator of the weak nucleon current is given by

$$\Gamma_\lambda = \gamma_4 \left(g_V \gamma_\lambda + \frac{g_M}{2M} \sigma_{\lambda\rho} k_\rho - g_A \gamma_\lambda \gamma_5 - i \frac{g_P}{m} k_\lambda \gamma_5 \right). \tag{8}$$

It involves the matrices $\sigma_{\lambda\rho} = (\gamma_\lambda\gamma_\rho - \gamma_\rho\gamma_\lambda)/2i$, the muon mass m, and the 4-momentum transfer

$$k_\lambda = \nu_\lambda - \mu_\lambda = (\mathbf{k}_\nu, -i\omega), \tag{9}$$

where ν_λ and μ_λ are the 4-momenta of the muon and neutrino, respectively. The form factors of vector interaction g_V, axial-vector interaction g_A, weak magnetism g_M, and induced pseudoscalar interaction g_P depend on $k^2 = k_\lambda k_\lambda$.

To go to the non-relativistic description of intranuclear nucleon we perform the Foldy–Wouthuysen transformation of relativistic Hamiltonian. Keeping the terms up to second-order in $1/M$ we obtain for non-relativistic Hamiltonian for j-th nucleon the following expression

$$\hat{H} = M + \frac{\hat{\mathbf{p}}_j^2}{2M} + U(r_j) + \frac{\Delta U(r_j)}{8M^2} + \frac{U'(r_j)}{4M}(\boldsymbol{\sigma}[\mathbf{n_j} \times \frac{\hat{\mathbf{p}}_j}{M}]) + \hat{h}_j(\sigma_\mu, \sigma_\nu) e^{-i\omega t}, \tag{10}$$

$$\begin{aligned} \hat{h}_j(\sigma_\mu, \sigma_\nu) = \frac{G\cos\theta_C}{\sqrt{2}} e^{-i\mathbf{k}_\nu \mathbf{r}_j} \Bigg\{ & i j_4(\sigma_\mu, \sigma_\nu) \Bigg[G'_V + G'_P(\boldsymbol{\sigma}_j \mathbf{n}_\nu) + \\ & + g'_A(\boldsymbol{\sigma}_j \frac{\hat{\mathbf{p}}_j}{M}) + iG_1(\mathbf{n}_\nu[\boldsymbol{\sigma}_j \times \frac{\hat{\mathbf{p}}_j}{M}]) + G_2(\mathbf{n}_\nu \frac{\hat{\mathbf{p}}_j}{M}) + i g_P \frac{U'(r_j)}{4M^2}(\boldsymbol{\sigma}_j \mathbf{n}_j) \Bigg] + \\ & + \mathbf{j}(\sigma_\mu, \sigma_\nu) \Bigg[G'_A \boldsymbol{\sigma}_j + g'_V \frac{\hat{\mathbf{p}}_j}{M} + i g_1 [\boldsymbol{\sigma}_j \times \frac{\hat{\mathbf{p}}_j}{M}] + \\ & + g_2 \left[[\boldsymbol{\sigma} \times \frac{\hat{\mathbf{p}}_j}{M}] \times \frac{\hat{\mathbf{p}}_j}{M} \right] + g_3 \boldsymbol{\sigma}_j (\mathbf{n}_\nu \frac{\hat{\mathbf{p}}_j}{M}) + g_V \frac{U'(r_j)}{4M^2} [\boldsymbol{\sigma}_j \times \mathbf{n}_j] \Bigg] \Bigg\} \hat{\tau}_-(j), \end{aligned} \tag{11}$$

where $\mathbf{n}_\nu = \mathbf{k}_\nu / k_\nu$, $\mathbf{n}_j = \mathbf{r}_j / r_j$, and

$$\begin{aligned} G'_V &= g_V \left(1 + \varepsilon - \frac{m}{4M}\varepsilon \right) - g_M \frac{m}{2M}\varepsilon, \\ G'_A &= g_A \left(1 - \frac{\varepsilon^2}{2} \right) - \varepsilon \left(g_V (1 - \frac{\eta}{2}) + g_M (1 - \eta) \right), \\ G'_P &= \varepsilon \left((g_P - g_A - g_V)(1 - \frac{\eta}{2}) - g_M(1 - \eta) \right), \\ g'_V &= g_V - \frac{\varepsilon}{2} g_A, \qquad g'_A = g_A (1 + \frac{\varepsilon}{2}) + g_P \frac{\eta}{2}, \\ G_1 &= \frac{\varepsilon}{2}(g_V + g_A + 2g_M), \qquad G_2 = -\frac{\varepsilon}{2} g_A, \\ g_1 &= \frac{1}{2}(\varepsilon g_A - \eta(g_V + 2g_M)), \qquad g_2 = \frac{1}{2} g_A, \qquad g_3 = -\frac{\varepsilon}{2} g_A, \end{aligned} \tag{12}$$

$$\varepsilon = \frac{E_\nu}{2M}, \qquad \eta = \frac{\omega}{2M}. \tag{13}$$

The non-relativistic Hamiltonian of muon capture by free nucleon (i.e. $U(r) = 0$) with second-order corrections was earlier obtained in Ref.[13][1] as well as in the other forms in Refs.[14, 11].

[1]Our result differs by the factor 1/4 in the term $m/(4M)\varepsilon$ in the definition of G'_V and by the sign of g_3.

The zero-order and first-order terms are proportional to the operators $\hat{1}$, $\boldsymbol{\sigma}_j$, $\hat{\mathbf{p}}_j$, and $(\boldsymbol{\sigma}_j \hat{\mathbf{p}}_j)$. The second-order terms involve the additional set of operators: $[\boldsymbol{\sigma}_j \times \hat{\mathbf{p}}_j]$, $[[\boldsymbol{\sigma}_j \times \hat{\mathbf{p}}_j] \times \hat{\mathbf{p}}_j]$, $\boldsymbol{\sigma}_j(\mathbf{n}_\nu \hat{\mathbf{p}}_j)$, $(\boldsymbol{\sigma}_j \mathbf{n}_j)$, and $[\boldsymbol{\sigma}_j \times \mathbf{n}_j]$.

3 Neutrino angular distribution

Let $|J_f M_f\rangle$ be the wave function that describes the internal state of the final nucleus with spin J_f and its projection M_f onto the z axis. At the same time, the initial state of the mesic atom with total angular momentum F involving a nucleus with spin J_i and a muon is represented as

$$|F\rangle = \sum_\xi a_\xi(F) \sum_{M_i \sigma_\mu} C^{F\xi}_{J_i M_i s_\mu \sigma_\mu} |J_i M_i\rangle \psi_\mu(\sigma_\mu). \tag{14}$$

Polarization and alignment of the ensemble of the mesic atoms with given total angular momentum F are described by density matrix

$$\rho_{\xi\xi'}(F) = \overline{a_\xi(F) a^*_{\xi'}(F)}, \qquad \sum_\xi \rho_{\xi\xi}(F) = 1, \tag{15}$$

or by spin-tensors

$$\tau_{Qq}(F) = \sum_{\xi\xi'} C^{F\xi'}_{F\xi Qq} \rho_{\xi\xi'}(F), \qquad \tau_{00}(F) = 1. \tag{16}$$

The energy of the neutrino emitted in the fixed transition $|J_i\rangle \to |J_f\rangle$ is

$$E_\nu = M_f \left[\left(1 + \frac{2Q_\mu}{M_f}\right)^{1/2} - 1 \right] \simeq Q_\mu \left(1 - \frac{Q_\mu}{2M_f} + \ldots \right), \tag{17}$$

where $Q_\mu = M_i + E_\mu - M_f$, M_i and M_f are the masses of the initial and final nuclei, respectively. Using the Fermi rule, we find that the differential probability of muon capture per unit time from a given state $|F\rangle$ of the hyperfine structure has the form

$$\begin{aligned} \frac{dw^F(\mathbf{n}_\nu)}{d\Omega} &= \frac{1}{(2\pi)^2} \times \\ &\times \sum_{\sigma_\nu M_f} \left| \sum_\xi a_\xi(F) \sum_{M_i \sigma_\mu} C^{F\xi}_{J_i M_i s_\mu \sigma_\mu} \langle J_f M_f | \sum_{j=1}^{A} \hat{h}_j(\sigma_\mu, \sigma_\nu) | J_i M_i \rangle \right|^2 \frac{E_\nu^2}{1 + E_\nu/M_f}. \end{aligned} \tag{18}$$

It is convenient to introduce the multipole expansions for the matrix elements of all operators appearing in the non-relativistic Hamiltonian. Generalizing the definitions of Ref.[6] for the operators of the first-order Hamiltonian we get for scalar operators

$$\begin{aligned} \langle J_f M_f | \sum_{j=1}^{A} e^{-i\mathbf{k}_\nu \mathbf{r}_j} \left\{ \begin{array}{c} \hat{1} \\ (\boldsymbol{\sigma}_j \hat{\mathbf{p}}_j) \\ iU'(r_j)(\boldsymbol{\sigma}_j \mathbf{n}_j) \end{array} \right\} \hat{\tau}_-(j) | J_i M_i \rangle = \\ = (4\pi)^{3/2} \sum_{um} (-1)^u Y^*_{um}(\mathbf{n}_\nu) C^{J_f M_f}_{J_i M_i um} \left\{ \begin{array}{c} [0uu] \\ [0uu, p] \\ [0uu, r] \end{array} \right\}, \end{aligned} \tag{19}$$

and for q-th spherical components of vector operators

$$\begin{aligned}
&\langle J_f M_f | \sum_{j=1}^{A} e^{-i\mathbf{k}_\nu \mathbf{r}_j} \left\{ \begin{array}{c} \sigma_{jq} \\ \hat{p}_{jq} \\ i[\boldsymbol{\sigma}_j \times \hat{\mathbf{p}}_j]_q \\ [[\boldsymbol{\sigma}_j \times \hat{\mathbf{p}}_j] \times \hat{\mathbf{p}}_j]_q \\ U'(r_j)[\boldsymbol{\sigma}_j \times \mathbf{n}_j]_q \\ \sigma_{jq}(\mathbf{n}_\nu \hat{\mathbf{p}}_j) \end{array} \right\} \hat{\tau}_-(j) | J_i M_i \rangle = \\
&= \frac{(4\pi)^{3/2}}{\sqrt{3}} \sum_{wm} (-1)^w Y^*_{wm}(\mathbf{n}_\nu) \sum_{uM} C^{uM}_{1qwm} C^{J_f M_f}_{J_i M_i uM} \left\{ \begin{array}{c} [1wu] \\ [1wu, p] \\ [1wu, \sigma p] \\ [1wu, \sigma p^2] \\ [1wu, r] \\ \{1wu, \sigma p\} \end{array} \right\}.
\end{aligned} \tag{20}$$

The quantity $\{1wu, \sigma p\}$ is linear combination of the reduced matrix elements $[kwu, \sigma p]$ ($k = 0, 1$ or 2)

$$\begin{aligned}
\{1wu, \sigma p\} = (-1)^{w-u} \sum_k (-1)^k \sqrt{\frac{2k+1}{6(2u+1)}} \times \\
\times \left(\sqrt{w}\, U(w\, w{-}1\, 1k, 1u)[k\, w{-}1\, u, \sigma p] - \right. \\
\left. - \sqrt{w+1}\, U(w\, w{+}1\, 1k, 1u)[k\, w{+}1\, u, \sigma p] \right),
\end{aligned} \tag{21}$$

defined by

$$\begin{aligned}
C^{J_f M_f}_{J_i M_i uM}[kwu, \sigma p] = \sqrt{2}\, \langle J_j M_f | \sqrt{\frac{3}{4\pi}} \sum_{j=1}^{A} j_w(k_\nu r_j) \times \\
\times \sum_{nm} C^{uM}_{knwm} i^w Y_{wm}(\mathbf{r}_j) \left(\sum_{\lambda q} C^{kn}_{1\lambda 1q} \sigma_{j\lambda} \hat{p}_{jq} \right) \hat{\tau}_-(j) | J_i M_i \rangle.
\end{aligned} \tag{22}$$

Note that $[0uu, \sigma p] = -\sqrt{2}[0uu, p]$. We use the normalized Racah function $U(abcd, ef) = \sqrt{(2e+1)(2f+1)}\, W(abcd, ef)$. All reduced matrix elements are real-valued quantities, provided that the nuclear wave functions are transformed under time reversal in standard way [15]

$$\hat{T} | JM \rangle = (-1)^{J+M} | J \, -M \rangle. \tag{23}$$

Let $\mathbf{n}_\mu$ be the unit vector along the axis z of orientation of the ensemble of the initial mesic atoms with given spin F. Polarization and alignment of this ensemble are fixed by spin-tensors $\tau_{10}(F)$ and $\tau_{20}(F)$ or normalized parameters

$$p_1(F) = \sqrt{\frac{F+1}{F}} \tau_{10}(F), \qquad p_2(F) = \sqrt{\frac{(F+1)(2F+3)}{F(2F-1)}} \tau_{20}(F). \tag{24}$$

Both parameters are equal unity if only the state corresponding to the maximal projection $\xi = F$ is populated ($p_1(F) = \langle \xi \rangle / F$ is the conventional polarization).

After straightforward calculation we get from Eq.(18) the general expression for neu-

trino angular distribution

$$\frac{dw^F(\mathbf{n}_\nu)}{d\Omega} = \frac{C_\mu}{4\pi}\frac{2J_f+1}{2J_i+1}\sum_K(2K+1)\tau_{K0}(F)P_K(\mathbf{n}_\nu\mathbf{n}_\mu)\times$$
$$\times\Bigg\{U(\frac{1}{2}J_iFK,FJ_i)\sum_{uu'}U(J_fu'J_iK,J_iu)\Bigg(C^{u'0}_{u0K0}V(u)V(u')-$$
$$-\frac{2}{\sqrt{3}}C^{u'0}_{u0K0}\sum_w C^{w0}_{u010}A(wu)V(u')+$$
$$+\frac{1}{3}(-1)^{u'-u}\sum_{ww'}(-1)^{w'-w}C^{w'0}_{w0K0}U(1w'uK,u'w)A(wu)A(w'u')+$$
$$+\frac{\sqrt{2}}{3}\sum_{ww'N}\sqrt{3(2N+1)(2u'+1)(2w+1)}\begin{Bmatrix} w' & u' & 1\\ w & u & 1\\ N & K & 1\end{Bmatrix}C^{w'0}_{w0N0}C^{N0}_{10K0}\times$$
$$\times A(wu)A(w'u')\Bigg)+$$
$$+\sum_N\sqrt{6(2N+1)(2J_i+1)(2F+1)}\begin{Bmatrix} J_i & F & 1/2\\ J_i & F & 1/2\\ N & K & 1\end{Bmatrix}\sum_{uu'}U(J_fu'J_iN,J_iu)\times \tag{25}$$
$$\times\Bigg(C^{N0}_{K010}C^{u'0}_{u0N0}V(u)V(u')-\frac{2}{\sqrt{3}}\sum_w C^{u'0}_{w0K0}U(u1u'K,wN)A(wu)V(u')-$$
$$-2\sqrt{\frac{2}{3}}\sum_w A(wu)V(u')\sum_\Lambda C^{u'0}_{w0\Lambda0}C^{\Lambda0}_{K010}U(u1u'\Lambda,wN)U(11NK,1\Lambda)-$$
$$-\frac{1}{3}(-1)^{u'-u}C^{N0}_{K010}\sum_{ww'}(-1)^{w'-w}C^{w'0}_{w0N0}U(1w'uN,u'w)A(wu)A(w'u')+$$
$$+\frac{2}{3}(-1)^N\sum_{ww'}\sqrt{\frac{2w'+1}{2w+1}}C^{u'0}_{w'010}C^{w0}_{u'0K0}U(u1u'K,wN)A(wu)A(w'u')+$$
$$+\frac{\sqrt{2}}{3}\sum_{ww'}C^{w'0}_{w0K0}\sqrt{3(2N+1)(2u'+1)(2w+1)}\begin{Bmatrix} u & w & 1\\ u' & w' & 1\\ N & K & 1\end{Bmatrix}\times$$
$$\times A(wu)A(w'u')\Bigg)\Bigg\}.$$

It is a series in the Legendre polynomials $P_0(\cos\theta)=1$, $P_1(\cos\theta)=cos\theta$, $P_2(\cos\theta)=3(\cos^2\theta-1)/2\ldots$, where θ is the angle between $\mathbf{n}_\nu$ and $\mathbf{n}_\mu$. The constant is

$$C_\mu = 8(G\cos\theta_C)^2\left(\frac{mZe^2}{1+m/M_i}\right)^3\frac{R(Z)E_\nu^2}{1+E_\nu/M_f}, \tag{26}$$

where Z is the charge of the initial nucleus, and $R(Z)$ is the correction factor for its non-pointlikeness. The amplitudes $V(u)$ and $A(wu)$ for the second-order Hamiltonian are given by

$$V(u)=\begin{cases} G'_V[0uu]+G_1\dfrac{\{1u,\sigma p\}}{M}+G_2\dfrac{\{1u,p\}}{M}, & \text{if } \pi_i(-1)^u=\pi_f,\\ G'_P\{1u\}+g'_A\dfrac{[0uu,p]}{M}+g_P\dfrac{[0uu,r]}{4M^2}, & \text{if } \pi_i(-1)^u=-\pi_f,\end{cases} \tag{27}$$

$$A(wu) = \begin{cases} G'_A[1wu] + g_2\dfrac{[1wu,\sigma p^2]}{M^2} + g_3\dfrac{\{1wu,\sigma p\}}{M}, & \text{if } \pi_i(-1)^w = \pi_f, \\ g'_V\dfrac{[1wu,p]}{M} + g_1\dfrac{[1wu,\sigma p]}{M} + g_V\dfrac{[1wu,r]}{4M^2}, & \text{if } \pi_i(-1)^w = -\pi_f, \end{cases} \tag{28}$$

where

$$\begin{aligned} \left\{ \begin{array}{c} \{1u\} \\ \{1u,p\} \\ \{1u,\sigma p\} \end{array} \right\} = -\sqrt{\frac{u}{3(2u+1)}} \left\{ \begin{array}{c} [1\,u{-}1\,u] \\ [1\,u{-}1\,u,p] \\ [1\,u{-}1\,u,\sigma p] \end{array} \right\} + \\ + \sqrt{\frac{u+1}{3(2u+1)}} \left\{ \begin{array}{c} [1\,u{+}1\,u] \\ [1\,u{+}1\,u,p] \\ [1\,u{+}1\,u,\sigma p] \end{array} \right\}, \end{aligned} \tag{29}$$

π_i and π_f are parities of initial and final nuclear states, respectively.

The case of transition $J_i \to J_f = J_i \pm 1$, $\pi_i = \pi_f$ with dominating Gamow-Teller matrix element [101] was considered in Ref.[12]. The neutrino angular distribution has the form

$$\begin{aligned} \frac{dw^F(\mathbf{n}_\nu)}{d\Omega} = \frac{C_\mu[101]^2}{12\pi}\frac{2J_f+1}{2J_i+1}\Big(a_0 + a_1 p_1(F)(\mathbf{n}_\nu\mathbf{n}_\mu) + \\ + a_2(F)p_2(F)\frac{3(\mathbf{n}_\nu\mathbf{n}_\mu)^2-1}{2}\Big), \end{aligned} \tag{30}$$

where

$$a_0 = G'_A(G'_A - \frac{2}{3}G'_P)C_1(J_i,J_f,F), \tag{31}$$

$$a_1 = -G'^2_A C_2(J_i,J_f,F) - G'_A(G'_A - G'_P)C_3(J_i,J_f,F), \tag{32}$$

$$a_2 = G'_A G'_P C_4(J_i,J_f,F). \tag{33}$$

The explicit expressions for coefficients $C_i(J_i,J_f,F)$ are presented in Ref.[12]. Due to high sensitivity of the anisotropy a_2 to G'_P the form factor g_P can be measured by this way. Note that this effect is of first-order type ($G'_A G'_P \sim \varepsilon$). Using the general expression (25) it is easy to estimate the contribution of non-leading matrix elements for any given transition.

4 Hyperfine effect

The rate of muon capture from hyperfine state $|F_\pm\rangle$ ($F_\pm = J_i \pm 1/2$) of mesic atom is given by isotropic term of differential probability (25), i.e.

$$\Lambda_\pm = \oint \left(\frac{dw^{F_\pm}(\mathbf{n}_\nu)}{d\Omega}\right) d\Omega. \tag{34}$$

It can be represented in the form proposed in Ref.[8]

$$\Lambda_F = \overline{\Lambda} + \delta\Lambda_F, \tag{35}$$

where the statistically-averaged muon capture rate is given by

$$\overline{\Lambda} = \frac{C_\mu}{2}\frac{2J_f+1}{2J_i+1}\sum_u \left(x^2(u) + y^2(u)\right), \tag{36}$$

and the hyperfine increment takes the form

$$\begin{aligned}\delta\Lambda_F = {} & \frac{C_\mu}{2}\cdot\frac{2J_f+1}{2J_i+1}\cdot\frac{J_i(J_i+1)+3/4-F(F+1)}{2J_i(J_i+1)}\times \\ & \times\sum_u\Bigg((J_i(J_i+1)+u(u+1)-J_f(J_f+1))\left(\frac{x^2(u)}{u}-\frac{y^2(u)}{u+1}\right)- \\ & -2\sqrt{(J_i+J_f+u+2)(J_i-J_f+u+1)(J_i+J_f-u)(J_f-J_i+u+1)}\times \\ & \times\frac{x(u+1)y(u)}{u+1}\Bigg),\end{aligned} \tag{37}$$

where

$$x(u) = \sqrt{\frac{2u}{2u+1}}V(u) - \sqrt{\frac{2(u+1)}{3(2u+1)}}A(uu) + \sqrt{\frac{2}{3}}A(u-1\,u), \tag{38}$$

$$y(u) = \sqrt{\frac{2(u+1)}{2u+1}}V(u) + \sqrt{\frac{2u}{3(2u+1)}}A(uu) - \sqrt{\frac{2}{3}}A(u+1\,u). \tag{39}$$

However, the muon capture rates $\Lambda_\pm$ may be brought also to the form

$$\begin{aligned}\Lambda_+ = {} & \frac{C_\mu}{2}\,\frac{2J_f+1}{2J_i+1}\,\frac{1}{2(J_i+1)}\sum_u\frac{1}{u+1}\times \\ & \times\Big(\sqrt{(J_i+J_f-u)(J_f-J_i+u+1)}x(u+1)+ \\ & +\sqrt{(J_i+J_f+u+2)(J_i-J_f+u+1)}y(u)\Big)^2,\end{aligned} \tag{40}$$

$$\begin{aligned}\Lambda_- = {} & \frac{C_\mu}{2}\,\frac{2J_f+1}{2J_i+1}\,\frac{1}{2J_i}\sum_u\frac{1}{u+1}\times \\ & \times\Big(\sqrt{(J_i+J_f+u+2)(J_i-J_f+u+1)}x(u+1)- \\ & -\sqrt{(J_i+J_f-u)(J_f-J_i+u+1)}y(u)\Big)^2.\end{aligned} \tag{41}$$

Putting

$$x(u) = \begin{cases} M_u(u), & \text{if } \pi_i(-1)^u = \pi_f, \\ -M_u(-u), & \text{if } \pi_i(-1)^u = -\pi_f, \end{cases} \tag{42}$$

$$y(u) = \begin{cases} M_u(-u-1), & \text{if } \pi_i(-1)^u = \pi_f, \\ M_u(u+1), & \text{if } \pi_i(-1)^u = -\pi_f. \end{cases} \tag{43}$$

we get the formulas [5] for muon capture rates from different hyperfine states expressed in terms of the amplitudes $M_u(k)$.

On the other hand, using Eqs.(27), (28) we obtain by this way the explicit expressions

for the amplitudes $M_u(k)$ with the accuracy up to the second-order terms in $1/M$

$$\begin{aligned}
M_u(u) = \sqrt{\frac{2}{2u+1}} \Bigg\{ & \sqrt{u}\, G'_V[0uu] - \sqrt{\frac{u+1}{3}}\, G'_A[1uu] + \\
& + \sqrt{\frac{2u+1}{3}} \left(g'_V - \frac{u}{2u+1} G_2 \right) \frac{[1u{-}1\,u,p]}{M} + \sqrt{\frac{u(u+1)}{3(2u+1)}}\, G_2\, \frac{[1u{+}1\,u,p]}{M} - \\
& - \sqrt{\frac{2u+1}{3}} \left(\frac{u}{2u+1} G_1 - g_1 - \frac{u+1}{2(2u+1)} g_3 \right) \frac{[1u{-}1\,u,\sigma p]}{M} + \\
& + \sqrt{\frac{u(u+1)}{3(2u+1)}} \left(G_1 + \frac{1}{2} g_3 \right) \frac{[1u{+}1\,u,\sigma p]}{M} + \\
& + \frac{1}{2} \sqrt{\frac{(u-1)(u+1)}{3(2u+1)}} g_3 \frac{[2u{-}1\,u,\sigma p]}{M} - \frac{1}{2} \sqrt{\frac{(u+1)(u+2)}{3(2u+1)}} g_3 \frac{[2u{+}1\,u,\sigma p]}{M} - \\
& - \sqrt{\frac{u+1}{3}}\, g_2\, \frac{[1uu,\sigma p^2]}{M^2} + \sqrt{\frac{2u+1}{3}}\, g_V\, \frac{[1u{-}1\,u,r]}{4M^2} \Bigg\},
\end{aligned} \tag{44}$$

$$\begin{aligned}
M_u(-u-1) = \sqrt{\frac{2}{2u+1}} \Bigg\{ & \sqrt{u+1}\, G'_V[0uu] + \sqrt{\frac{u}{3}}\, G'_A[1uu] - \\
& - \sqrt{\frac{2u+1}{3}} \left(g'_V - \frac{u+1}{2u+1} G_2 \right) \frac{[1u{+}1\,u,p]}{M} - \sqrt{\frac{u(u+1)}{3(2u+1)}}\, G_2\, \frac{[1u{-}1\,u,p]}{M} + \\
& + \sqrt{\frac{2u+1}{3}} \left(\frac{u+1}{2u+1} G_1 - g_1 - \frac{u}{2(2u+1)} g_3 \right) \frac{[1u{+}1\,u,\sigma p]}{M} - \\
& - \sqrt{\frac{u(u+1)}{3(2u+1)}} \left(G_1 + \frac{1}{2} g_3 \right) \frac{[1u{-}1\,u,\sigma p]}{M} - \\
& - \frac{1}{2} \sqrt{\frac{u(u-1)}{3(2u+1)}}\, g_3\, \frac{[2u{-}1\,u,\sigma p]}{M} + \frac{1}{2} \sqrt{\frac{u(u+2)}{3(2u+1)}}\, g_3\, \frac{[2u{+}1\,u,\sigma p]}{M} + \\
& + \sqrt{\frac{u}{3}}\, g_2\, \frac{[1uu,\sigma p^2]}{M^2} - \sqrt{\frac{2u+1}{3}}\, g_V\, \frac{[1u{+}1\,u,r]}{4M^2} \Bigg\},
\end{aligned} \tag{45}$$

$$\begin{aligned}
M_u(-u) = \sqrt{\frac{2}{2u+1}} \Bigg\{ & -\sqrt{\frac{2u+1}{3}} \left(G'_A - \frac{u}{2u+1} G'_P \right) [1u{-}1\,u] - \\
& - \sqrt{\frac{u(u+1)}{3(2u+1)}}\, G'_P\, [1u{+}1\,u] - \sqrt{u} \left(g'_A + \frac{1}{3} g_3 \right) \frac{[0uu,p]}{M} + \\
& + \sqrt{\frac{u+1}{3}}\, g'_V\, \frac{[1uu,p]}{M} + \sqrt{\frac{u+1}{3}} \left(g_1 + \frac{1}{2} g_3 \right) \frac{[1uu,\sigma p]}{M} + \\
& + \sqrt{\frac{(u-1)(2u+1)}{6(2u-1)}} g_3 \frac{[2u{-}2\,u,\sigma p]}{M} - \frac{1}{6} \sqrt{\frac{(u+1)(2u+3)}{2u-1}} g_3 \frac{[2uu,\sigma p]}{M} - \\
& - \sqrt{\frac{2u+1}{3}}\, g_2\, \frac{[1u{-}1\,u,\sigma p^2]}{M^2} - \sqrt{u}\, g_P\, \frac{[0uu,r]}{4M^2} + \sqrt{\frac{u+1}{3}}\, g_V\, \frac{[1uu,r]}{4M^2} \Bigg\},
\end{aligned} \tag{46}$$

$$
\begin{aligned}
M_u(u+1) = \sqrt{\frac{2}{2u+1}}\Bigg\{ & -\sqrt{\frac{2u+1}{3}}\left(G'_A - \frac{u+1}{2u+1}G'_P\right)[1u{+}1\,u] - \\
& - \sqrt{\frac{u(u+1)}{3(2u+1)}}\,G'_P\,[1u{-}1\,u] + \sqrt{u+1}\left(g'_A - \frac{1}{3}g_3\right)\frac{[0uu,p]}{M} + \\
& + \sqrt{\frac{u}{3}}\,g'_V\,\frac{[1uu,p]}{M} + \sqrt{\frac{u}{3}}\left(g_1 + \frac{1}{2}g_3\right)\frac{[1uu,\sigma p]}{M} - \\
- \sqrt{\frac{(u+2)(2u+1)}{6(2u+3)}}\,g_3\,\frac{[2u{+}2\,u,\sigma p]}{M} & + \frac{1}{6}\sqrt{\frac{u(2u-1)}{2u+3}}\,g_3\,\frac{[2uu,\sigma p]}{M} - \\
- \sqrt{\frac{2u+1}{3}}\,g_2\,\frac{[1u{+}1\,u,\sigma p^2]}{M^2} & + \sqrt{u+1}\,g_P\,\frac{[0uu,r]}{4M^2} + \sqrt{\frac{u}{3}}\,g_V\,\frac{[1uu,r]}{4M^2}\Bigg\}.
\end{aligned} \tag{47}
$$

We see that the second-order terms do not change the general expressions (35)-(37) or (40), (41) for muon capture rates from hyperfine sublevels but only are added to the known zero-order and first-order terms in the amplitudes $M_u(k)$.

To study the magnitude of interference between the zero-order and second-order terms with respect to the squared first-order terms let us consider as an example a transition $3/2 \to 1/2$ between the states with the same parity. The muon capture rates are given by

$$
\Lambda_+ = \frac{3C_\mu}{8}\left(M_1^2(2) + \frac{2}{\sqrt{15}}M_1(2)M_2(2) + \frac{1}{15}M_2^2(2) + \frac{16}{15}M_2^2(-3)\right), \tag{48}
$$

$$
\Lambda_- = \frac{2C_\mu}{3}\left(M_1^2(-1) + \frac{1}{16}M_1^2(2) - \frac{\sqrt{15}}{8}M_1(2)M_2(2) + \frac{15}{16}M_2^2(2)\right), \tag{49}
$$

The first-order term $\sim G'_P[101]$ may exceed in the amplitude $M_1(2)$ both the zero-order term $\sim (G'_A - 2G'_P/3)[121]$ and the other first-order velocity terms due to the dominance of Gamow–Teller matrix element [101], on the one hand, and the high absolute value of the form factor g_P, on the other hand. Just in this case the squared first-order term $\sim G'^2_P$ appears in the numerator of the ratio (1). Then it is apparent that the significant influence on the magnitude of the squared amplitude $M_1(2)$ is beyond the capabilities of any second-order term in it, as

$$
\begin{aligned}
M_1(2) = \sqrt{\frac{2}{3}}\Bigg\{ & -\frac{\sqrt{2}}{3}\,G'_P\,[101] - \left(G'_A - \frac{2}{3}G'_P\right)[121] + \\
& + \sqrt{2}\left(g'_A - \frac{1}{3}g_3\right)\frac{[011,p]}{M} + \frac{1}{\sqrt{3}}\,g'_V\,\frac{[111,p]}{M} + \\
+ \frac{1}{\sqrt{3}}\left(g_1 + \frac{1}{2}g_3\right)\frac{[111,\sigma p]}{M} & + \frac{1}{6\sqrt{5}}\,g_3\,\frac{[211,\sigma p]}{M} - \sqrt{\frac{3}{10}}\,g_3\,\frac{[231,\sigma p]}{M} - \\
& - g_2\,\frac{[121,\sigma p^2]}{M^2} + \sqrt{2}\,g_P\,\frac{[011,r]}{4M^2} + \frac{1}{\sqrt{3}}\,g_V\,\frac{[111,r]}{4M^2}\Bigg\}.
\end{aligned} \tag{50}
$$

It is clear that the contribution of second-order terms does not exceed several percents. However, the terms associated with nucleon-nucleus potential, which were never considered earlier, deserve special attention.

5 Summary

The second-order in $1/M$ non-relativistic Hamiltonian of muon capture is obtained. For the first time the terms associated with nucleon-nucleus potential are considered. General treatment of the hyperfine effect in muon capture rate is given with taking into account of the second-order terms. In particular, the generalized expressions for the usually used amplitudes $M_u(k)$ are presented. It is shown that in the situation of the dominance of the squared first-order terms the interference of zero-order and second-order terms has no significant influence. However, the second-order corrections may be of interest due to their sensitivity to the nuclear potential.

General expression for neutrino (recoil nucleus) angular distribution in muon capture by nucleus with non-zero spin is proposed. In the approximation of dominating Gamow–Teller matrix element we discuss the high sensitivity to g_P of the term related with alignment of the initial mesic atom. Such alignment may be realized in capture of muons by oriented atoms. Using the general formula for the angular distribution the contribution of non-leading matrix elements to alignment effect can be easy estimated for any given transition.

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